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Now, $\tan^2 x \sec^2 x - \tan^2 x$

Let $\tan x = t$

$\sec^2 x = \frac{dt}{dx}$

$\therefore \frac{t^2 \cdot \cancel{\sec^2 x} \cdot dt}{\cancel{\sec^2 x}} = t^2$

$= \cancel{t^2 dt - \frac{t^2 dt}{\sec^2 x}}$

$= \cancel{\tan} \tan^2 x \cdot \sec^2 x - \tan^2 x$

$= t^2 \cdot \frac{dt \cdot \cancel{\sec^2 x}}{\cancel{\sec^2 x}} - (\sec^2 x - 1)\, dx$

$= t^2 dt - (\sec^2 x - 1)\, dx$

On integrating we get,

$= \frac{t^3}{3} - \tan x - x$

$= \frac{\tan^3 x}{3} - \tan x - x + C$ Ans

Q) Integrate $\int \frac{d\theta}{\sin\theta \cdot \cos^3\theta}$

$\frac{\sin\theta}{\cos^3\theta} + \frac{1}{\sin\theta \cdot \cos\theta}$

Solution: Given,

$= \frac{d\theta}{\sin\theta \cdot \cos^3\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos^3\theta}$

$= \frac{\sin^2\theta}{\sin\theta \cdot \cos^3\theta} + \frac{\cancel{\cos^2\theta}}{\sin\theta \cdot \cancel{\cos^3\theta}}$

$= \frac{\sin\theta}{\cos^3\theta} + \frac{1}{\sin\theta \cdot \cos\theta}$

$= I_1 + I_2$

Let $I_1 = \frac{\sin\theta}{\cos^3\theta}$

Let $\cos\theta = t$

$-\sin\theta = \frac{dt}{dx}$

$\therefore dx = \frac{-dt}{\sin\theta}$

$I_1 = \frac{-\sin\theta}{t^3} \cdot \frac{dt}{\sin\theta}$

On integrating, $= -\frac{1}{t^3} dt$

$= \frac{+t^{-3+1}}{+2}$

$= \frac{1}{2} \cdot \frac{1}{t^2}$

$= \frac{1}{2}\cos^2\theta$

$I_2 = \frac{1}{\sin\theta \cdot \cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}$

$= \tan\theta + \cot\theta$

On integrating we get

$= \log\sec\theta + \log\sin\theta$

$= \log \sec\theta \cdot \sin\theta$

$= \log(\tan\theta)$

$= \frac{1}{2}\sec^2\theta + \log\tan\theta$. Ans

Q Integrate $\int \frac{\sqrt{\tan x}\, dx}{\sin x \cdot \cos x}$.

Solution,

Given, $y = \int \frac{\sqrt{\tan x}}{\sin x \cos x}$

Ordinarily on the both sides by ~~cos~~ $\sec^2 x$ we get

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$$y = \frac{\sqrt{\tan x}\cdot \sec^2 x}{\sin x \cdot \cos x \cdot \sec^2 x}$$

$$= \frac{\sqrt{\tan x}\cdot \sec^2 x}{\frac{\sin x}{\cos x}\cdot\left(\frac{\cos^2 x}{\cos^2 x}\right)}$$

$$= \frac{\sqrt{\tan x}\cdot \sec^2 x}{\tan x}$$

Now, let $\tan x = t$

$$\sec^2 x = \frac{dt}{dx}$$

$$\therefore dx = \frac{dt}{\sec^2 x}$$

$$\therefore = \frac{\sqrt{t}\cdot \cancel{\sec^2 x}}{t} \times \frac{dt}{\cancel{\sec^2 x}}$$

On integrating we get

$$= \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

$$= 2\, t^{1/2}$$

$$\therefore 2\sqrt{\tan x} + C \quad \text{Ans}$$

Q) $y = \frac{e^{2x}}{(1+e^x)} = \frac{e^x \cdot e^x}{(1+e^x)}$

Solution, put $1+e^x = t$

$$e^x = \frac{dt}{dx}$$

$$\therefore dx = \frac{dt}{e^x}$$

$$= \frac{\cancel{e^x}\, e^x}{t} \times \frac{dt}{\cancel{e^x}}$$

$$\int \frac{(t-1)}{t}\,dt$$

$$1 - \frac{1}{t}$$

$$= t - \log t$$

$$= 1+e^x - \log(1+e^x)$$

Ans

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V) $\int \frac{1}{e^x + e^{-x}} dx$

Solution, given $\frac{1}{e^x + e^{-x}}$

$= \frac{e^x}{(1+e^{2x})}$

put $e^x = t$

$e^x = \frac{dt}{dx}$

$\therefore dx = \frac{dt}{e^x}$

$\therefore I = \frac{e^x}{(1+t^2)} \cdot \frac{dt}{e^x}$

$= \frac{dt}{1+t^2}$

$= \tan^{-1} t$ $\quad [\therefore \tan^{-1} x = \frac{1}{1+x^2}]$

$= \tan^{-1} e^x + c$ Ans

Q) $\int \frac{\sin^5 x}{\sqrt{\cos x}} dx$

Solution: Given $I = \int \frac{\sin^5 x}{\sqrt{\cos x}} dx$

$= \frac{\sin^2 x \cdot \sin^2 x \cdot \sin x}{\sqrt{\cos x}} dx$

Now, Let $\cos x = t$

$-\sin x = \frac{dt}{dx}$

$\therefore dx = \frac{-dt}{\sin x}$

$= -\frac{\sin^2 x \cdot \sin^2 x \cdot \cancel{\sin x}}{\sqrt{t}} \cdot \frac{dt}{\cancel{\sin x}}$

$= \frac{-\sin^2 x\,(1-\cos^2 x)}{\sqrt{t}} = \frac{(1-\cos^2 x)(1-\cos^2 x)}{\sqrt{t}}$

$= -\frac{(1-t^2)(1-t^2)}{\sqrt{t}}$

$= \frac{(1-t^2)^2}{\sqrt{t}} = -\left[\frac{1-2t^2+t^4}{\sqrt{t}}\right]$

$= -\left[t^{-\frac{1}{2}} - 2\,t^{3/2} + t^{7/2}\,dt\right]$

Now on integrating we get

$= 2\,t^{\frac{1}{2}} - \frac{4}{5}\,t^{5/2} + \frac{2}{9}\,t^{9/2}$

$= 2\,\cancel{\cos}$

$= 2\,t^{1/2}\left[1 - \frac{2}{5}\,t^{5/2}\right.$

$= -\left[2\,t^{1/2} - 2\cdot\frac{2}{5}\,t^{5/2} + \frac{2}{9}\,t^{9/2}\right]$

$= -2\,t^{1/2}\left[1 - \frac{2}{5}\,t^2 + \frac{1}{9}\,t^4\right]$

$= -2\sqrt{\cos}\left[1 - \frac{2}{5}\cos^2 x + \frac{1}{9}\cos^4 x\right]$

Ans